

Maximally supersymmetric solutions of R^2 supergravity

Sergei M. Kuzenko

*School of Physics M013, The University of Western Australia
35 Stirling Highway, Crawley W.A. 6009, Australia*

`sergei.kuzenko@uwa.edu.au`

Abstract

There are five maximally supersymmetric backgrounds in four-dimensional off-shell $\mathcal{N} = 1$ supergravity, two of which are well known: Minkowski superspace $\mathbb{M}^{4|4}$ and anti-de Sitter superspace $\text{AdS}^{4|4}$. The three remaining supermanifolds support spacetimes of different topology, which are: $\mathbb{R} \times S^3$, $\text{AdS}_3 \times \mathbb{R}$, and a supersymmetric plane wave isometric to the Nappi-Witten group. As is well known, the Minkowski and anti-de Sitter superspaces are solutions of the Poincaré and anti-de Sitter supergravity theories, respectively. Here we demonstrate that the other three superspaces are solutions of no-scale R^2 supergravity. We also present a new (probably the simplest) derivation of the maximally supersymmetric backgrounds of off-shell $\mathcal{N} = 1$ supergravity.

1 Introduction

There exist only five maximally supersymmetric backgrounds in off-shell $\mathcal{N} = 1$ supergravity in four dimensions. As purely bosonic backgrounds, the complete list was given by Festuccia and Seiberg [1]. Their results were re-derived in [2] using the superspace formalism developed in the mid-1990s [3] (see [4] for a review). As curved $\mathcal{N} = 1$ superspaces, all these backgrounds were described in [5]. The algebraic aspects of these backgrounds have recently been studied in [6].

We now list all maximally supersymmetric backgrounds of $\mathcal{N} = 1$ supergravity following [5].¹ The simplest and most well-known is Minkowski superspace $\mathbb{M}^{4|4}$ [7, 8]. It is characterised by the algebra of covariant derivatives

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} = -2i\mathcal{D}_{\alpha\dot{\beta}} , \quad (1.1a)$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0 , \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0 , \quad (1.1b)$$

$$[\mathcal{D}_a, \mathcal{D}_B] = 0 . \quad (1.1c)$$

The second oldest background is anti-de Sitter (AdS) superspace $\text{AdS}^{4|4}$ [9, 10, 11]. It is characterised by the algebra of covariant derivatives

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} = -2i\mathcal{D}_{\alpha\dot{\beta}} , \quad (1.2a)$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{R}M_{\alpha\beta} , \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 4R\bar{M}_{\dot{\alpha}\dot{\beta}} , \quad (1.2b)$$

$$[\mathcal{D}_a, \mathcal{D}_\beta] = -\frac{i}{2}\bar{R}(\sigma_a)_{\beta\dot{\gamma}}\bar{\mathcal{D}}^{\dot{\gamma}} , \quad [\mathcal{D}_a, \bar{\mathcal{D}}_{\dot{\beta}}] = \frac{i}{2}R(\sigma_a)_{\gamma\dot{\beta}}\mathcal{D}^\gamma , \quad (1.2c)$$

$$[\mathcal{D}_a, \mathcal{D}_b] = -|R|^2 M_{ab} , \quad (1.2d)$$

with $R = \text{const.}$ The Riemann tensor of AdS^4 may be deduced from (1.2d) to be

$$\mathfrak{R}_{abcd} = -|R|^2(\eta_{ac}\eta_{bd} - \eta_{ad}\eta_{bc}) . \quad (1.3)$$

The three remaining superspaces are characterised by formally identical anti-commutation relations [5]

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0 , \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 0 , \quad \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\beta}}\} = -2i\mathcal{D}_{\alpha\dot{\beta}} , \quad (1.4a)$$

$$[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot{\beta}}] = i\varepsilon_{\alpha\beta}G^{\gamma\dot{\beta}}\mathcal{D}_\gamma , \quad [\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -i\varepsilon_{\dot{\alpha}\dot{\beta}}G_{\beta}^{\gamma\dot{\gamma}}\bar{\mathcal{D}}_{\dot{\gamma}} , \quad (1.4b)$$

¹In all cases, the superspace covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}})$ have the form $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}) = E_A^M \partial_M + \frac{1}{2}\Omega_A^{bc} M_{bc}$, where M_{bc} is the Lorentz generator. In the case of Minkowski superspace, one can choose the Lorentz connection Ω_A^{bc} to vanish, and the inverse vielbein E_A^M to have the Akulov-Volkov form [7].

$$[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -i\varepsilon_{\dot{\alpha}\dot{\beta}} G_{\beta}{}^{\dot{\gamma}} \mathcal{D}_{\alpha\dot{\gamma}} + i\varepsilon_{\alpha\beta} G^{\gamma}{}_{\dot{\beta}} \mathcal{D}_{\gamma\dot{\alpha}} , \quad (1.4c)$$

where G_b is covariantly constant,

$$\mathcal{D}_A G_b = 0 . \quad (1.4d)$$

The difference between these superspaces is encoded in the Lorentzian type of G_a . Since $G^2 = G^a G_a$ is constant, the geometry (1.4) describes three different superspaces, $\mathbb{M}_T^{4|4}$, $\mathbb{M}_S^{4|4}$ and $\mathbb{M}_N^{4|4}$, which correspond to the choices $G^2 < 0$, $G^2 > 0$ and $G^2 = 0$, respectively. The Lorentzian manifolds, which are the bosonic bodies of the superspaces $\mathbb{M}_T^{4|4}$, $\mathbb{M}_S^{4|4}$ and $\mathbb{M}_N^{4|4}$, are $\mathbb{R} \times S^3$, $\text{AdS}_3 \times \mathbb{R}$ and a pp-wave spacetime,² respectively. The Riemann curvature tensor of these spacetimes is

$$\mathfrak{R}_{abcd} = \frac{1}{4} \left\{ G_c (G_a \eta_{bd} - G_b \eta_{ad}) - G_d (B_a \eta_{bc} - G_b \eta_{ac}) - G^2 (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}) \right\} . \quad (1.5)$$

The superspace $\mathbb{M}_T^{4|4}$ is the universal covering of $\mathcal{M}^{4|4} = \text{SU}(2|1)$. The bosonic body of $\mathcal{M}^{4|4}$ is $\text{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$. The isometry group of $\mathcal{M}^{4|4}$ is $\text{SU}(2|1) \times \text{U}(2)$. One can think of $\mathbb{M}_T^{4|4}$ as a supersymmetric extension of Einstein's static universe. $\mathcal{N} = 1$ supersymmetric field theories on $\mathbb{R} \times S^3$ were studied in the mid-1980s by Sen [13]. The superspace $\mathbb{M}_S^{4|4}$ is the universal covering of $\widetilde{\mathcal{M}}^{4|4} = \text{SU}(1, 1|1)$. The bosonic body of $\widetilde{\mathcal{M}}^{4|4}$ is $\text{U}(1, 1) = (\text{AdS}_3 \times S^1)/\mathbb{Z}_2$. The isometry group of $\widetilde{\mathcal{M}}^{4|4}$ is $\text{SU}(1, 1|1) \times \text{U}(2)$.

The superspace (1.2) is a maximally supersymmetric solution of anti-de Sitter supergravity described by the action (see, e.g., [3] for a review)

$$S_{\text{SUGRA}} = -\frac{3}{\kappa^2} \int d^4 x d^2 \theta d^2 \bar{\theta} E + \left\{ \frac{\mu}{\kappa^2} \int d^4 x d^2 \theta \mathcal{E} + \text{c.c.} \right\} , \quad (1.6)$$

where κ is the gravitational coupling constant and μ a cosmological parameter. The integration measures E and \mathcal{E} in (1.6) correspond to the full superspace and its chiral subspace, respectively. The equations of motion corresponding to (1.6) are

$$G_a = 0 , \quad R = \mu , \quad (1.7)$$

see [3] for a pedagogical derivation. Setting $\mu = 0$ in (1.6) gives the action for $\mathcal{N} = 1$ Poincaré supergravity [14]. Minkowski superspace (1.1) is a maximally supersymmetric solution of this theory.

In this note, we are going to show that the superspaces (1.4) are maximally supersymmetric solutions of scale-invariant R^2 supergravity

$$S = \alpha \int d^4 x d^2 \theta d^2 \bar{\theta} E R \bar{R} + \left\{ \beta \int d^4 x d^2 \theta \mathcal{E} R^3 + \text{c.c.} \right\}$$

²The latter spacetime was shown in [6] to be isometric to the Nappi-Witten group NW_4 [12].

$$= \int d^4x d^2\theta d^2\bar{\theta} E \left\{ \alpha R \bar{R} + (\beta R^2 + \bar{\beta} \bar{R}^2) \right\} , \quad (1.8)$$

with α and β a real and a complex dimensionless parameters, respectively. The α -term in (1.8) is generated as a one-loop quantum correction in $\mathcal{N} = 1$ supersymmetric field theories coupled to supergravity [15, 16, 17]. The component structure of this term was described in [18]. Although the β -term in (1.8) breaks the $U(1)$ R -symmetry, adding such a contribution to the α -term is completely natural, keeping in mind that a massless covariantly chiral scalar superfield Φ , $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$, is described in supergravity by an action

$$S_{\text{matter}} = \int d^4x d^2\theta d^2\bar{\theta} E \left\{ \Phi \bar{\Phi} + \frac{1}{2} \xi (\Phi^2 + \bar{\Phi}^2) \right\} , \quad (1.9)$$

with ξ a dimensionless parameter. The choice $\xi = 0$ corresponds to the conformal scalar multiplet model which is dual to the improved tensor multiplet [19]. Another natural choice is $\xi = 1$ and corresponds to a non-conformal scalar multiplet which is dual to the free tensor multiplet model [20].

The higher-derivative supergravity model (1.8) has recently been studied in [21].³ Along with the supergravity action, both terms in (1.8) have also been discussed in the framework of supersymmetric models for inflation, see [22, 23] and references therein.

This note is organised as follows. In section 2 we briefly discuss the various superspace formulations for $\mathcal{N} = 1$ conformal supergravity, and then present a new derivation of the maximally supersymmetric backgrounds of off-shell $\mathcal{N} = 1$ supergravity. In section 3 we prove that the curved superspaces described by (1.4) are solutions of the no-scale supergravity model (1.8). Some concluding comments are given in section 4.

2 A new derivation of the maximally supersymmetric backgrounds in off-shell $\mathcal{N} = 1$ supergravity

Every off-shell formulation for $\mathcal{N} = 1$ supergravity can be described using the superspace geometry pioneered by Howe thirty five years ago [24] and soon after reviewed

³Action (1.8) can be rewritten in a manifestly super-Weyl invariant form, as in [21], by introducing a chiral compensator ϕ , $\bar{\mathcal{D}}_{\dot{\alpha}}\phi = 0$, and replacing R with the super-Weyl invariant chiral scalar $\mathbb{R} = -\frac{1}{4}\phi^{-2}(\bar{\mathcal{D}}^2 - 4R)\phi$ and the full superspace measure E with $E\phi\bar{\phi}$. Such a superconformal reformulation is sometimes useful, in particular for the component reduction, however it does not offer new insights to the analysis in this note.

and further developed in [25]. This curved superspace geometry is based on the structure group $\text{SL}(2, \mathbb{C}) \times \text{U}(1)$, and nowadays it is often referred to as $\text{U}(1)$ superspace. The algebra of supergravity covariant derivatives is as follows:

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\} = -2i\mathcal{D}_{\alpha\dot{\alpha}} , \quad (2.1a)$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar{R}M_{\alpha\beta} , \quad \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} = 4R\bar{M}_{\dot{\alpha}\dot{\beta}} , \quad (2.1b)$$

$$\begin{aligned} [\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot{\beta}}] &= i\varepsilon_{\alpha\beta} \left(\bar{R} \bar{\mathcal{D}}_{\dot{\beta}} + G^\gamma_{\dot{\beta}} \mathcal{D}_\gamma - (\mathcal{D}^\gamma G^\delta_{\dot{\beta}}) M_{\gamma\delta} + 2\bar{W}_{\dot{\beta}}^{\dot{\gamma}\dot{\delta}} \bar{M}_{\dot{\gamma}\dot{\delta}} \right) \\ &\quad + i(\bar{\mathcal{D}}_{\dot{\beta}} \bar{R}) M_{\alpha\beta} - \frac{i}{3} \varepsilon_{\alpha\beta} \bar{X}^{\dot{\gamma}} \bar{M}_{\dot{\gamma}\dot{\beta}} - \frac{i}{2} \varepsilon_{\alpha\beta} \bar{X}_{\dot{\beta}} \mathbb{J} . \end{aligned} \quad (2.1c)$$

Here the $\text{U}(1)_R$ generator \mathbb{J} is normalised by

$$[\mathbb{J}, \mathcal{D}_\alpha] = \mathcal{D}_\alpha , \quad [\mathbb{J}, \bar{\mathcal{D}}_{\dot{\alpha}}] = -\bar{\mathcal{D}}_{\dot{\alpha}} . \quad (2.2)$$

The torsion superfields R , $G_{\alpha\dot{\alpha}}$, $W_{\alpha\beta\gamma}$, and X_α obey the Bianchi identities:

$$\bar{\mathcal{D}}_{\dot{\alpha}} R = 0 , \quad \bar{\mathcal{D}}_{\dot{\alpha}} X_\alpha = 0 , \quad \bar{\mathcal{D}}_{\dot{\alpha}} W_{\alpha\beta\gamma} = 0 , \quad (2.3a)$$

$$X_\alpha = \nabla_\alpha R - \bar{\mathcal{D}}^{\dot{\alpha}} G_{\alpha\dot{\alpha}} , \quad \mathcal{D}^\alpha X_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}} . \quad (2.3b)$$

The reason why the superspace geometry defined by (2.1) is adequate to describe $\mathcal{N} = 1$ conformal supergravity is the fact that the algebra (2.1) does not change under a super-Weyl transformation

$$\mathcal{D}'_\alpha = e^{\frac{1}{2}L} \left(\mathcal{D}_\alpha + 2(\mathcal{D}^\beta L) M_{\beta\alpha} - \frac{3}{2}(\mathcal{D}_\alpha) L \mathbb{J} \right) \quad (2.4)$$

accompanied by induced transformations of the torsion superfields. The parameter L in (2.4) is a real unconstrained superfield.

Before turning to the derivation of the maximally supersymmetric backgrounds of supergravity, it is worth commenting on other superspace approaches to describe $\mathcal{N} = 1$ conformal supergravity. The $\text{U}(1)$ superspace of [24] is a gauge fixed version of 4D $\mathcal{N} = 1$ conformal superspace [26], in which the entire superconformal algebra $\text{SU}(2, 2|1)$ is gauged in superspace (see also [27] for a review of the relationship between the $\text{U}(1)$ and conformal superspaces). When studying supersymmetric backgrounds of supergravity, it suffices to work with $\text{U}(1)$ superspace, and therefore we do not use conformal superspace in this note.

The superspace geometry developed by Grimm, Wess and Zumino [28] is obtained from (2.1) by setting

$$X_\alpha = 0 . \quad (2.5)$$

Under this condition, the $U(1)_R$ connection can be gauged away and the structure group reduces to $SL(2, \mathbb{C})$. Requirement (2.5) can always be achieved by applying a specially chosen super-Weyl transformation (2.4). If such a super-Weyl gauge is chosen, one stays with a residual super-Weyl plus $U(1)$ gauge freedom given by [29]

$$\mathcal{D}'_\alpha = e^{\bar{\sigma} - \sigma/2} \left(\mathcal{D}_\alpha + (\mathcal{D}^\beta \sigma) M_{\alpha\beta} \right), \quad \bar{\mathcal{D}}_{\dot{\alpha}} \sigma = 0. \quad (2.6)$$

As is well known (see, e.g., [25] for a review), the different off-shell formulations for $\mathcal{N} = 1$ supergravity are obtained by coupling conformal supergravity (described, e.g. using $U(1)$ superspace) to a compensator. The latter is a chiral scalar in the case of the old minimal formulation [14, 30], a real linear superfield for the new minimal formulation [31], and a complex linear superfield for the non-minimal formulation [32, 33]. Our analysis of maximally supersymmetric backgrounds of supergravity does not require fixing any specific compensator.

We now recall an important theorem concerning the maximally supersymmetric backgrounds [34, 4]. For any (off-shell) supergravity theory in D dimensions, all maximally supersymmetric spacetimes correspond to those supergravity backgrounds which are characterised by the following properties: (i) all Grassmann-odd components of the superspace torsion and curvature tensors vanish; and (ii) all Grassmann-even components of the torsion and curvature tensors are annihilated by the spinor derivatives. In the case of 4D $\mathcal{N} = 1$ supergravity, this theorem means the following:

$$X_\alpha = 0; \quad (2.7a)$$

$$W_{\alpha\beta\gamma} = 0; \quad (2.7b)$$

$$\mathcal{D}_\alpha R = 0 \longrightarrow \mathcal{D}_A R = 0; \quad (2.7c)$$

$$\mathcal{D}_\alpha G_{\beta\dot{\beta}} = \bar{\mathcal{D}}_{\dot{\alpha}} G_{\beta\dot{\beta}} = 0 \longrightarrow \mathcal{D}_A G_{\beta\dot{\beta}} = 0. \quad (2.7d)$$

Equation (2.7d) has an integrability condition that follows from (2.1b). It is

$$0 = \{\bar{\mathcal{D}}_{\dot{\alpha}}, \bar{\mathcal{D}}_{\dot{\beta}}\} G_{\gamma\dot{\gamma}} = 4R \bar{M}_{\dot{\alpha}\dot{\beta}} G_{\gamma\dot{\gamma}} = 2R(\varepsilon_{\dot{\gamma}\dot{\alpha}} G_{\gamma\dot{\beta}} + \varepsilon_{\dot{\gamma}\dot{\beta}} G_{\gamma\dot{\alpha}}), \quad (2.8)$$

and therefore

$$RG_{\alpha\dot{\alpha}} = 0. \quad (2.9)$$

Eq. (2.7a) tells us that all maximally supersymmetric backgrounds are realised in terms of the Grimm-Wess-Zumino superspace geometry [28].

Relation (2.9) (actually its θ -independent part) was given in [1] without derivation. Let us also show that (2.9) is a simple consequence of the general analysis given in section 6.4 of [3]. Consider a background superspace $(\mathcal{M}^{4|4}, \mathcal{D})$. A supervector field $\xi = \xi^B E_B = \xi^b E_b + \xi^\beta E_\beta + \bar{\xi}_{\dot{\beta}} \bar{E}^{\dot{\beta}}$ on $(\mathcal{M}^{4|4}, \mathcal{D})$ is called Killing if

$$\delta_{\mathcal{K}} \mathcal{D}_A = [\mathcal{K}, \mathcal{D}_A] = 0, \quad \mathcal{K} := \xi^B(z) \mathcal{D}_B + \frac{1}{2} K^{bc}(z) M_{bc} + i\tau(z) \mathbb{J}, \quad (2.10)$$

for some Lorentz (K^{bc}) and R -symmetry (τ) parameters. All parameters ξ^β , K^{bc} , τ are determined in terms of ξ^b ,

Let $\xi = \xi^A E_A$ be a conformal Killing supervector field of $(\mathcal{M}^{4|4}, \mathcal{D})$. As demonstrated in section 6.4 of [3], its explicit form is

$$\xi^A = \left(\xi^a, \xi^\alpha, \bar{\xi}_{\dot{\alpha}} \right) = \left(\xi^a, -\frac{i}{8} \bar{\mathcal{D}}_{\dot{\beta}} \xi^{\dot{\beta}\alpha}, -\frac{i}{8} \mathcal{D}^\beta \xi_{\beta\dot{\alpha}} \right), \quad (2.11)$$

where the vector component $\xi_{\alpha\dot{\alpha}}$ is real and obeys the equation [3]

$$\mathcal{D}_{(\alpha} \xi_{\beta)\dot{\beta}} = 0, \quad (2.12)$$

which implies

$$(\mathcal{D}^2 + 2\bar{R}) \xi_{\alpha\dot{\alpha}} = 0. \quad (2.13)$$

In accordance with (2.7d), $G_{\alpha\dot{\alpha}}$ is covariantly constant, and hence it is a solution of (2.12). Then (2.13) reduces to (2.9).

3 Maximally supersymmetric solutions of no-scale R^2 supergravity

We now prove that the curved superspaces described by (1.4) are solutions of the no-scale supergravity model (1.8). For this we will use the background-field method for $\mathcal{N} = 1$ supergravity as developed by Grisaru and Siegel [35] and further elaborated in [3].

We denote infinitesimal increments of the supergravity prepotentials by H^a and σ , where H^a is real unconstrained and σ is covariantly chiral, $\bar{\mathcal{D}}_{\dot{\alpha}} \sigma = 0$. The variations of various supergravity functionals under such an infinitesimal change in the prepotentials was computed in section 5.6 of the book [3] (see also [16]). The results we need here are:

$$\delta \int d^4x d^2\theta d^2\bar{\theta} E R \bar{R} = -\frac{1}{4} \int d^4x d^2\theta d^2\bar{\theta} E \left\{ \sigma \mathcal{D}^2 R + \bar{\sigma} \bar{\mathcal{D}}^2 \bar{R} \right\}$$

$$\begin{aligned}
& + \frac{1}{2} \int d^4x d^2\theta d^2\bar{\theta} E H^{\alpha\dot{\alpha}} \left\{ 2R\bar{R}G_{\alpha\dot{\alpha}} - \frac{1}{6}(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 \bar{R}) + \frac{i}{6}\mathcal{D}_{\alpha\dot{\alpha}}(\bar{\mathcal{D}}^2 \bar{R} - \mathcal{D}^2 R) \right. \\
& \quad \left. + \frac{2}{3}R \overset{\leftrightarrow}{\mathcal{D}}_{\alpha\dot{\alpha}} \bar{R} + \frac{1}{3}(\mathcal{D}_{\alpha} R)\bar{\mathcal{D}}_{\dot{\alpha}} \bar{R} \right\} , \tag{3.1a}
\end{aligned}$$

$$\begin{aligned}
& \delta \int d^4x d^2\theta d^2\bar{\theta} E R^2 = 3 \int d^4x d^2\theta d^2\bar{\theta} E (\sigma - \bar{\sigma}) R^2 \\
& + \int d^4x d^2\theta d^2\bar{\theta} E H^{\alpha\dot{\alpha}} \left\{ G_{\alpha\dot{\alpha}} - i\mathcal{D}_{\alpha\dot{\alpha}} \right\} R^2 . \tag{3.1b}
\end{aligned}$$

It is seen that both variations (3.1b) and (3.1b) vanish for the backgrounds (1.4). If the parameter β in (1.8) is non-zero, $\beta \neq 0$, the anti-de Sitter superspace (1.2) is not a solution of the equations of motion for (1.8).

In accordance with (2.7b), all maximally supersymmetric backgrounds of $\mathcal{N} = 1$ supergravity are conformally flat.⁴ Therefore all of them are solutions of the equations of motion for $\mathcal{N} = 1$ conformal supergravity described by the chiral action [37, 38]

$$I_{\text{CSG}} = \int d^4x d^2\theta \mathcal{E} W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} + \text{c.c.}$$

4 Concluding comments

It is instructive to compare the maximally supersymmetric backgrounds (1.2) and (1.4) with their counterparts for three-dimensional $\mathcal{N} = 2$ supergravity.

In three dimensions, the maximally supersymmetric backgrounds of off-shell $\mathcal{N} = 2$ supergravity were classified in [39], and also reviewed and elaborated in [4]. The three-dimensional analogue of (1.2) is the (1,1) AdS superspace [40]. The three-dimensional analogues of the backgrounds (1.4) are given by the following algebra of covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^\alpha)$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0 , \quad \{\bar{\mathcal{D}}_\alpha, \bar{\mathcal{D}}_\beta\} = 0 , \tag{4.1a}$$

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_\beta\} = -2i(\gamma^c)_{\alpha\beta} \left(\mathcal{D}_c - 2\mathcal{S}M_c - i\mathcal{C}_c \mathbb{J} \right) + 4\varepsilon_{\alpha\beta} \left(\mathcal{C}^c M_c - i\mathcal{S} \mathbb{J} \right) , \tag{4.1b}$$

$$[\mathcal{D}_a, \mathcal{D}_\beta] = i\varepsilon_{abc}(\gamma^b)_\beta{}^\gamma \mathcal{C}^c \mathcal{D}_\gamma + (\gamma_a)_\beta{}^\gamma \mathcal{S} \mathcal{D}_\gamma , \tag{4.1c}$$

$$[\mathcal{D}_a, \bar{\mathcal{D}}_\beta] = -i\varepsilon_{abc}(\gamma^b)_\beta{}^\gamma \mathcal{C}^c \bar{\mathcal{D}}_\gamma + (\gamma_a)_\beta{}^\gamma \mathcal{S} \bar{\mathcal{D}}_\gamma , \tag{4.1d}$$

$$[\mathcal{D}_a, \mathcal{D}_b] = 4\varepsilon_{abc} \left(\mathcal{C}^c \mathcal{C}_d + \delta^c_d \mathcal{S}^2 \right) M^d . \tag{4.1e}$$

Here M_c denotes the Lorentz generator (defined in [39]) and the $U(1)_R$ generator \mathbb{J} is defined similarly to (2.2). The scalar \mathcal{S} and vector \mathcal{G}_b components of the torsion tensor

⁴This is not true for some maximally supersymmetric backgrounds of $\mathcal{N} = 2$ supergravity [36].

are constrained by

$$\mathcal{D}_A \mathcal{S} = 0, \quad \mathcal{D}_\alpha \mathcal{C}_b = 0 \quad \implies \quad \mathcal{D}_a \mathcal{C}_b = 2\varepsilon_{abc} \mathcal{C}^c \mathcal{S}, \quad (4.2)$$

and hence $\mathcal{C}^b \mathcal{C}_b = \text{const.}$ We point out that the solution with $\mathcal{C}_a = 0$ corresponds to the (2,0) AdS superspace [40]. However, here we are interested in the case $\mathcal{C}_b \neq 0$. When both \mathcal{S} and \mathcal{C}_b are non-vanishing, the above curved superspace is a maximally supersymmetric solution of topologically massive type II supergravity [4]. In the case $\mathcal{S} = 0$ and $\mathcal{C}_b \neq 0$, the above superspace is a solution of three-dimensional R^2 supergravity [41].

One of the most interesting properties of the maximally supersymmetric backgrounds (1.4) is that they allow for the Maxwell-Goldstone multiplet models which describe partial $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking [5] and reduce to the Bagger-Galperin model [42] in the flat limit, $G_a \rightarrow 0$.

The $\mathcal{N} = 2$ analogue of scale-invariant R^2 supergravity (1.8) was given in [43]. It is of interest to see which rigid $\mathcal{N} = 2$ maximally supersymmetric backgrounds [36] are solutions of this theory.

Acknowledgements:

It is my pleasure to acknowledge the hospitality of Dima Sorokin and the INFN, Sezione di Padova, where this project was designed. I also thank Luca Martucci for asking a question that provided the rationale for writing up the construction described in this paper. Daniel Butter is gratefully acknowledged for helpful correspondence. Joseph Novak is gratefully acknowledged for comments on the manuscript. This work is supported in part by the Australian Research Council, project No. DP160103633.

References

- [1] G. Festuccia and N. Seiberg, “Rigid supersymmetric theories in curved superspace,” JHEP **1106**, 114 (2011) [arXiv:1105.0689 [hep-th]].
- [2] S. M. Kuzenko, “Symmetries of curved superspace,” JHEP **1303**, 024 (2013) [arXiv:1212.6179 [hep-th]].
- [3] I. L. Buchbinder and S. M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace*, IOP, Bristol, 1995 (Revised Edition: 1998).
- [4] S. M. Kuzenko, “Supersymmetric spacetimes from curved superspace,” PoS CORFU **2014**, 140 (2015) [arXiv:1504.08114 [hep-th]].

- [5] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Nilpotent chiral superfield in N=2 supergravity and partial rigid supersymmetry breaking,” *JHEP* **1603**, 092 (2016) [arXiv:1512.01964 [hep-th]].
- [6] P. de Medeiros, J. Figueroa-O’Farrill and A. Santi, “Killing superalgebras for Lorentzian four-manifolds,” arXiv:1605.00881 [hep-th].
- [7] V. P. Akulov and D. V. Volkov, “Goldstone fields with spin 1/2,” *Theor. Math. Phys.* **18**, 28 (1974) [*Teor. Mat. Fiz.* **18**, 39 (1974)].
- [8] A. Salam and J. A. Strathdee, “Super-gauge transformations,” *Nucl. Phys. B* **76**, 477 (1974).
- [9] B. W. Keck, “An alternative class of supersymmetries,” *J. Phys. A* **8**, 1819 (1975).
- [10] B. Zumino, “Nonlinear realization of supersymmetry in de Sitter space,” *Nucl. Phys. B* **127**, 189 (1977).
- [11] E. A. Ivanov and A. S. Sorin, “Superfield formulation of OSp(1,4) supersymmetry,” *J. Phys. A* **13**, 1159 (1980).
- [12] C. R. Nappi and E. Witten, “A WZW model based on a nonsemisimple group,” *Phys. Rev. Lett.* **71**, 3751 (1993) [hep-th/9310112].
- [13] D. Sen, “Supersymmetry in the space-time $R \times S^3$,” *Nucl. Phys. B* **284**, 201 (1987).
- [14] J. Wess and B. Zumino, “Superfield Lagrangian for supergravity,” *Phys. Lett. B* **74**, 51 (1978).
- [15] I. L. Buchbinder and S. M. Kuzenko, “Matter superfields in external supergravity: Green functions, effective action and superconformal anomalies,” *Nucl. Phys. B* **274**, 653 (1986).
- [16] I. L. Buchbinder and S. M. Kuzenko, “Quantization of the classically equivalent theories in the superspace of simple supergravity and quantum equivalence,” *Nucl. Phys. B* **308**, 162 (1988).
- [17] I. L. Buchbinder and S. M. Kuzenko, “Nonlocal action for supertrace anomalies in superspace of N=1 supergravity,” *Phys. Lett. B* **202**, 233 (1988).
- [18] S. Theisen, “Fourth-order supergravity,” *Nucl. Phys. B* **263**, 687 (1986) Addendum: [*Nucl. Phys. B* **269**, 744 (1986)].
- [19] B. de Wit and M. Roček, “Improved tensor multiplets,” *Phys. Lett. B* **109**, 439 (1982).
- [20] W. Siegel, “Gauge spinor superfield as a scalar multiplet,” *Phys. Lett. B* **85**, 333 (1979).
- [21] S. Ferrara, A. Kehagias and M. Porrati, “ \mathcal{R}^2 supergravity,” *JHEP* **1508**, 001 (2015) [arXiv:1506.01566 [hep-th]].
- [22] S. V. Ketov and A. A. Starobinsky, “Embedding $(R + R^2)$ -inflation into supergravity,” *Phys. Rev. D* **83**, 063512 (2011) [arXiv:1011.0240 [hep-th]].
- [23] S. Ferrara, R. Kallosh and A. Van Proeyen, “On the supersymmetric completion of $R + R^2$ gravity and cosmology,” *JHEP* **1311**, 134 (2013) [arXiv:1309.4052 [hep-th]].
- [24] P. S. Howe, “A superspace approach to extended conformal supergravity,” *Phys. Lett. B* **100**, 389 (1981); “Supergravity in superspace,” *Nucl. Phys. B* **199**, 309 (1982).
- [25] S. J. Gates Jr., M. T. Grisaru, M. Roček and W. Siegel, *Superspace, or One Thousand and One Lessons in Supersymmetry*, Benjamin/Cummings (Reading, MA), 1983, hep-th/0108200.

- [26] D. Butter, “N=1 conformal superspace in four dimensions,” *Annals Phys.* **325**, 1026 (2010) [arXiv:0906.4399 [hep-th]].
- [27] D. Butter and S. M. Kuzenko, “A dual formulation of supergravity-matter theories,” *Nucl. Phys. B* **854**, 1 (2012) [arXiv:1106.3038 [hep-th]].
- [28] R. Grimm, J. Wess and B. Zumino, “Consistency checks on the superspace formulation of supergravity,” *Phys. Lett. B* **73**, 415 (1978); “A complete solution of the Bianchi identities in superspace,” *Nucl. Phys. B* **152**, 255 (1979).
- [29] P. S. Howe and R. W. Tucker, “Scale invariance in superspace,” *Phys. Lett. B* **80**, 138 (1978).
- [30] K. S. Stelle and P. C. West, “Minimal auxiliary fields for supergravity,” *Phys. Lett. B* **74**, 330 (1978); S. Ferrara and P. van Nieuwenhuizen, “The auxiliary fields of supergravity,” *Phys. Lett. B* **74**, 333 (1978).
- [31] M. F. Sohnius and P. C. West, “An alternative minimal off-shell version of N=1 supergravity,” *Phys. Lett. B* **105**, 353 (1981).
- [32] P. Breitenlohner, “A geometric interpretation of local supersymmetry,” *Phys. Lett. B* **67**, 49 (1977); “Some invariant Lagrangians for local supersymmetry,” *Nucl. Phys.* **B124**, 500 (1977).
- [33] W. Siegel and S. J. Gates Jr. “Superfield supergravity,” *Nucl. Phys. B* **147**, 77 (1979).
- [34] S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Symmetries of curved superspace in five dimensions,” *JHEP* **1410**, 175 (2014) [arXiv:1406.0727 [hep-th]].
- [35] M. T. Grisaru and W. Siegel, “Supergraphity (I). Background field formalism,” *Nucl. Phys. B* **187**, 149 (1981); “Supergraphity (II). Manifestly covariant rules and higher loop finiteness,” *Nucl. Phys. B* **201**, 292 (1982).
- [36] D. Butter, G. Inverso and I. Lodato, “Rigid 4D $\mathcal{N} = 2$ supersymmetric backgrounds and actions,” *JHEP* **1509**, 088 (2015) [arXiv:1505.03500 [hep-th]].
- [37] W. Siegel, “Solution to constraints in Wess-Zumino supergravity formalism,” *Nucl. Phys. B* **142**, 301 (1978).
- [38] B. Zumino, “Supergravity and superspace,” in *Recent Developments in Gravitation - Cargèse 1978*, M. Lévy and S. Deser (Eds.), N.Y., Plenum Press, 1979, pp. 405–459.
- [39] S. M. Kuzenko, U. Lindström, M. Roček, I. Sachs and G. Tartaglino-Mazzucchelli, “Three-dimensional N=2 supergravity theories: From superspace to components,” *Phys. Rev. D* **89**, 085028 (2014) [arXiv:1312.4267 [hep-th]].
- [40] S. M. Kuzenko and G. Tartaglino-Mazzucchelli, “Three-dimensional N=2 (AdS) supergravity and associated supercurrents,” *JHEP* **1112**, 052 (2011).
- [41] S. M. Kuzenko, J. Novak and G. Tartaglino-Mazzucchelli, “Higher derivative couplings and massive supergravity in three dimensions,” *JHEP* **1509**, 081 (2015) [arXiv:1506.09063 [hep-th]].
- [42] J. Bagger and A. Galperin, “A new Goldstone multiplet for partially broken supersymmetry,” *Phys. Rev. D* **55**, 1091 (1997) [arXiv:hep-th/9608177].
- [43] S. M. Kuzenko and J. Novak, “On curvature squared terms in N=2 supergravity,” *Phys. Rev. D* **92**, no. 8, 085033 (2015) [arXiv:1507.04922 [hep-th]].